

AP Calculus Summer Work

This worksheet is due the second day of AP Calculus, Fall 2023. Work each problem completely. ALL ANSWERS SHOULD BE SUPPORTED BY APPROPRIATE WORK! Answers should be exact unless otherwise indicated. Expect a quiz over this material on the second day of class.

Factoring

Factor the following completely. If unfactorable, say the polynomial is prime.

1) $x^9 - x^6 - x^3 + 1$

2) $x^3 + 14x^2 + 45x$

3) $5x^2 + 29x + 20$

4) $8x^4 + 10x^2 - 3$

5) $x^3 - 4x^2 - 4x + 16$

Writing Equations of Lines

Write an equation of the line with the given description in the indicated form(s).

6) Passes through the points (-9, 3) and (5, 12)

Point-Slope:

Slope-Intercept:

7) Has x -intercept at 9 and y -intercept at 7

Slope-Intercept:

Standard:

- 8) Has slope of $-\frac{10}{13}$ and passes through the point $(-1, 5)$

Point-Slope:

Slope-Intercept:

- 9) is **parallel** to the line with equation $x = 4$ and passes through the point $(8, 1)$

- 10) is **perpendicular** to the line with equation $y = -2x + 5$ and passes through the point $(-7, 2)$

Point-Slope:

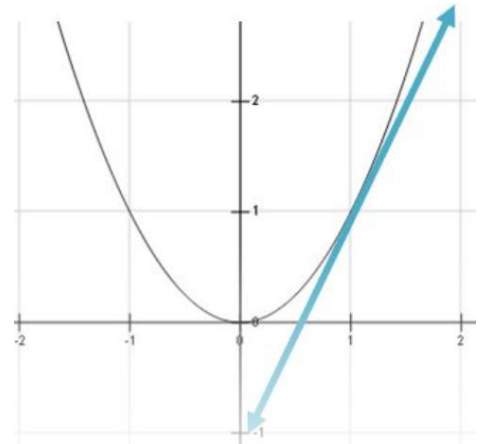
Slope-Intercept:

- 11) is **perpendicular** to the line whose slope is $\frac{5}{4}$ and passes through the point $(2, 1)$

Point-Slope:

Slope-Intercept:

- 12) Using the given graph, write an equation for the line shown.



- 13) Define a line **normal** to a curve at a point.

Domain

FIND THE DOMAIN OF THE FOLLOWING. EXPRESS ANSWERS IN INTERVAL NOTATION.

14) $f(x) = \frac{1}{x-3}$

15) $f(x) = \sqrt{x^2 - 2x - 15}$

16) $f(x) = \sqrt{\frac{5x+1}{8-x^2}}$

17) $f(x) = \frac{x^2-100}{x+10}$

18) $f(x) = 2^x - 9$

19) $f(x) = \sqrt{x^2 - 16}$

20) $f(x) = \frac{2x+15}{4x^2+4x-15}$

21) $f(x) = \ln(x + 12)$

22) $f(x) = \log_3 x - 4$

23) $f(x) = -\log(8 - x)$

Inverse Functions

- 24) Find $f^{-1}(x)$ if $f(x) = 5 - 4x$.
- 25) Find $f^{-1}(x)$ if $f(x) = \frac{7x-18}{-8x+15}$
- 26) Find $f^{-1}(x)$ if $f(x) = e^{x+2}$
- 27) Find $f^{-1}(x)$ if $f(x) = \ln x - 16$
- 28) Discuss the relationship between the domain and range of a function and its inverse.
- 29) Define a one-to-one function. Draw the graph of a one-to-one function.
- 30) $f(x) = \sin x$ is NOT a one-to-one function over its domain. Give an interval on which $f(x)$ would be considered one-to-one.

- 31) Discuss the relationship between the graph of a function and the graph of its inverse.
- 32) Suppose f is a one-to-one function and $f(9) = -18$. What is $f^{-1}(-18)$?
- 33) Given the one-to-one function f . The point (a, c) is on the graph of f . Give the coordinates of a point on the graph of f^{-1} .

Solving Equations

SOLVE THE FOLLOWING FOR THE REAL VALUES OF x . GIVE EXACT ANSWERS.

34) $\log_4(9x - 10) = \log_4 x$

35) $\log_5(2x - 7) = 3$

36) $5 - 2^{-x+1} = 0$

37) $\left(\frac{1}{9}\right)^{x+1} = 3^{2x+19}$

38) $0 = \frac{-12(x^2 + 3)^2 + 12x(2(x^2 + 3)(2x))}{(x^2 + 3)^4}$

39) $\ln(t^2 - 3t + 3) = 0$

40) $12x^2 - 92x + 120 = 0$

41) $\frac{10}{3}x^{-\frac{1}{3}} + \frac{5}{3}x^{\frac{2}{3}} = 0$

42) $(x + 4)^3 + x(3(x + 4)^2) = 0$

43) $\ln(x + 2) - \ln(4x + 3) = \ln\left(\frac{1}{x}\right)$

44) $\log_2 x + \log_2(x - 7) = 3$

Trigonometry

45) Solve for x given that $\cos x = -\frac{\sqrt{2}}{2}$, $0 \leq x < 2\pi$.

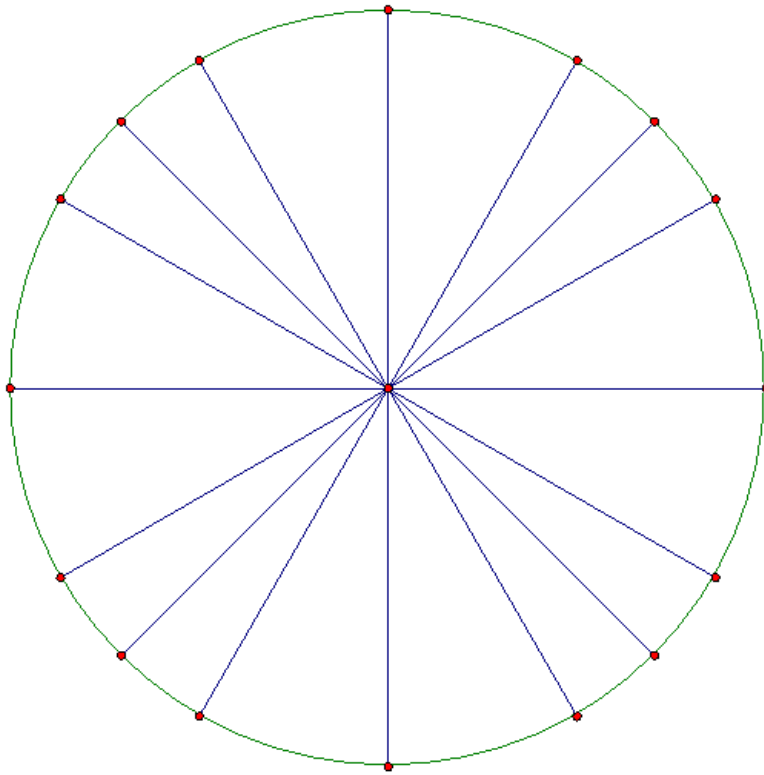
46) Give the general solution, in radians, for $\csc \theta = -2$.

47) For $0 \leq t \leq 6$, solve $2\sin\left(e^{\frac{t}{4}}\right) + 1 = 0$ for the exact value of t .

48) Solve $25\cos x + 25\cos^2 x - 25\sin^2 x = 0$ for $0 \leq x < 2\pi$

55) Complete the unit circle with the followin

- a) Degree measures that are multiples of 30° or 45°
- b) Equivalent radian measures
- c) Their sines, cosines, and tangents



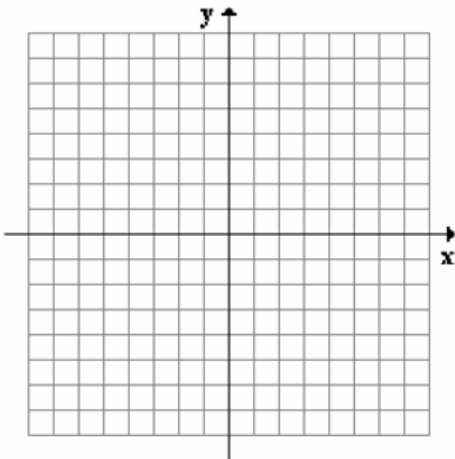
Conic Sections

- 56) Find the vertex, intercepts, and axis of symmetry for a parabola with equation $x = 2(y - 3)^2 + 4$.

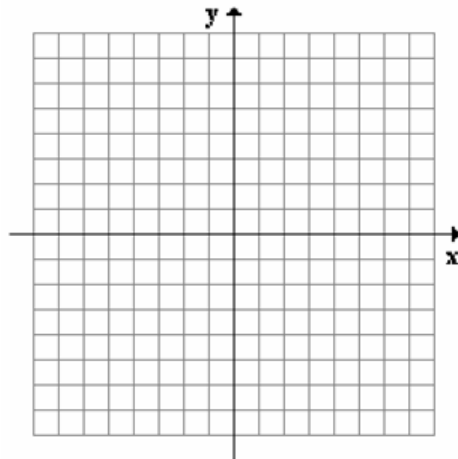
In which direction does this parabola open?

GRAPH THE FOLLOWING CONIC SECTIONS. INDICATE PERTINENT INFORMATION.

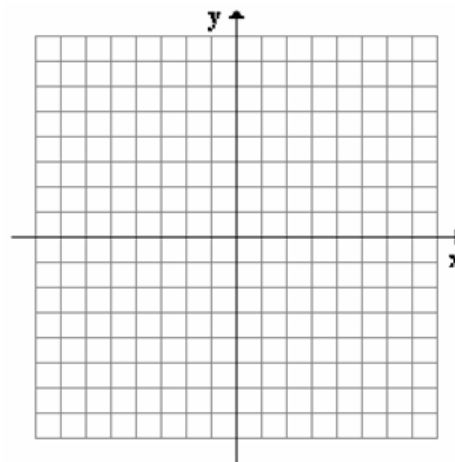
57) $x^2 + y^2 = 16$



58) $x - 3 = -(y - 2)^2$



59) $x = 4y - y^2$



Limits

GUESS THE VALUE OF THE LIMIT USING A TABLE OF VALUES.
CALCULATOR SHOULD BE IN RADIANS.

$$60) \quad \lim_{x \rightarrow 0} \frac{\sin^2 2x}{9x^2}$$

x			
$f(x)$			

x			
$f(x)$			

$$61) \quad \lim_{x \rightarrow 0} \frac{e^x - 1}{x}$$

x			
$f(x)$			

x			
$f(x)$			

$$62) \quad \lim_{h \rightarrow 0} \frac{e^{(2+h)} - e^2}{h}$$

h			
$f(h)$			

h			
$f(h)$			

EVALUATE THE FOLLOWING LIMITS. GIVE EXACT ANSWERS WHEN NECESSARY. **SHOW ALL ALGEBRAIC WORK.** WRITE OUT ALL NECESSARY LIMITS.

$$63) \quad \lim_{x \rightarrow 2^+} \frac{3x^2 + 10}{-2x + 4}$$

$$64) \quad \lim_{x \rightarrow 1} \sqrt{19x - 1}$$

$$65) \quad \lim_{x \rightarrow \infty} \frac{x^4 + 3x^2}{5x^3}$$

$$66) \quad \lim_{x \rightarrow -3} \frac{2x^2 - 18}{x + 3}$$

$$67) \quad \lim_{h \rightarrow 0} \frac{-4(x+h)^2 + 4x^2}{h}$$

$$68) \quad \lim_{h \rightarrow 0} \frac{\sqrt{x+h-1} - \sqrt{x-1}}{h}$$

$$69) \quad \lim_{x \rightarrow 2^+} \frac{x^2 + 12}{2-x}$$

$$70) \quad \lim_{x \rightarrow \infty} \frac{5x-18}{9-7x}$$

$$71) \quad \lim_{x \rightarrow a} \frac{3x^3 - 3a^3}{a-x}$$

$x \rightarrow a$
(a is a real number)

$$72) \quad \lim_{x \rightarrow k} \frac{x^{10} - k^{10}}{x^5 - k^5}$$

$x \rightarrow k$
(k is a real number)

$$73) \quad \lim_{x \rightarrow -2^+} \frac{x^2 + 2x}{\sqrt{x^2 + 4x + 4}}$$

$$74) \quad \lim_{x \rightarrow -\infty} f(x) \text{ given that } f(x) = \begin{cases} \frac{2x^2+7}{x^2} & \text{for } x > 10 \\ \frac{x^2}{2x^2+7} & \text{for } x \leq 10 \end{cases}$$

$$75) \quad \lim_{x \rightarrow \infty} f(x) \text{ given that } f(x) = \begin{cases} \frac{2x^2+7}{x^2} & \text{for } x > 10 \\ \frac{x^2}{2x^2+7} & \text{for } x \leq 10 \end{cases}$$

$$76) \quad \lim_{x \rightarrow 10} f(x) \text{ given that } f(x) = \begin{cases} \frac{2x^2+7}{x^2} & \text{for } x > 10 \\ \frac{x^2}{2x^2+7} & \text{for } x \leq 10 \end{cases}$$

Continuity

DETERMINE THE CONTINUITY OF THE FOLLOWING FUNCTIONS.

$$77) \quad f(x) = \begin{cases} 2x - 3 & \text{for } x > 1 \\ 5 & \text{for } x = 1 \\ -3x + 2 & \text{for } x < 1 \end{cases}$$

$$78) \quad f(x) = -4^x + x^4$$

$$79) \quad f(x) = \frac{1}{x^2-9} + \frac{1}{\sqrt{25-x^2}}$$

$$80) \quad f(x) = \tan x$$